Some New Topics in International Trade Theory

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(sheets: 16 and 29)

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Background for Ricardian trade theory

- **One of the oldest theories in economics**
  - Mercantilists (16-18th centuries)
  - Adam Smith (free trade)
  - Ricardo: Comparative advantage theory
  - Oppositions: Alexander Hamilton, Frederick List (German Historical School)

- **D. Ricardo’s theory of international trade**
  - Ricardo succeeded to explain gains from trade even in the case when a country has inferior production techniques than the other in all industries. (Absolute advantage vs. comparative advantage)
  - S. Ulam once asked S. if any economic theory is **true but not trivial**.
  - Samuelson’s answer: Ricardo’s theory of comparative advantage
A new interpretation in 2002-04.

- Faccarello (2015) Comparative Advantage

Comparative advantage vs. comparative cost

- Comparative advantage is not defined in a general case (with input trade).
- Cost comparison is still valid. Shiozawa (2016b)
Ricardian trade economy

Traditionally, dealt with cases

- $M$-country, $N$-commodity
  - Minimal model was $(2, 2)$ type.
- Production: labor input economy
- Capital goods.
  - Vertical integration
  - Applicable if no input goods are traded.

Explored in 1950’s.

- L. McKenzie, R. Jones
- Crucial defects: No input trade (intermediate goods)
Ricardo-Sraffa trade economy

- $M$-country, $N$-commodity case
- Production: material input
  - Capital: general name of input goods
  - Choice of production techniques
- Input trade (traded intermediate goods)
  - finished goods vs. intermediate goods
  - Introduction of trade in intermediate product necessitates a fundamental alteration of the theory.
  - Distinction between Ricardian t.e. and RS t.e. crucial.
  - Subtropical theory only applies to R.t.e.
Importance of RS trade economy

● The real RS t.e. is structurally different from R t.e.
  ■ A challenging problem for tropical theory.
  ■ N.B. If input goods are not traded, RS t.e. is reduced to R. t.e.

● Actual problems are related to RS t.e.
  ■ Industrial revolution in Lancashire, Cotton.
  ■ Fragmentation, Global value chain, etc.
Ricardian trade theory as subtropical convex geometry

- **Subtropical algebra**
  - $\mathbb{R}_+$
  - $a \oplus b = \min\{a, b\}$, $a \odot b = a \cdot b$
  - Isomorphic to the tropical (min, plus)-algebra in $\mathbb{R}$.
  - $\log: \mathbb{R}_+ \to \mathbb{R}$; $\log (a \cdot b) = \log (a) + \log (b)$.

- Well adapted to the description and analysis of R t.e.
- Details: Shiozawa (2012; 2015a)
- Many topics to be developed.
- Good concrete model of tropical geometry.
Bird’s-eye view of the theory

World pp set

Cephoid

Log-lifting over \( \Delta^{m-1} \times \Delta^{n-1} \)

Caley trick

Mixed subdivisions of \( n\Delta^{m-1} \)

McKenzie-Minabe diagram

World Production Frontier

Price Simplex \( \Delta^{n-1} \)

Covering Range Rp

Sharing Range Rw

Subtropical polytopes

Tropical polytopes

Tropical oriented matroid

Triangulations of \( \Delta^{m-1} \times \Delta^{n-1} \)

Tropical h-plane Arrangement

Fine type

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Ricardian trade economy: mathematical formulation

• Input coefficient matrix $A = (a_{ij})$
  - M-row N-column matrix
  - $a_{ij}$ labor input coefficient in country $i$ to produce product $j$

• Labor power $q = (q_i)$

• International value $v = (w, p)$
  - $w = (w_i)$ wage rate for country $i$
  - $p = (p_j)$ price for product $j$
Some notions: PPS, value, competitive pattern

- Production possibility set (PPS), a polytope in $\mathbb{R}_+^N$.
  $$\mathbf{P} = \{ \mathbf{y} \mid y_j = (\sum_i s_{ij}), \sum_j s_{ij} a_{ij} \leq q_i, s_{ij} \geq 0 \ \forall i \}$$

- $\mathbf{v} = (\mathbf{w}, \mathbf{p}) = (w_1, \ldots, w_M, p_1, \ldots, p_N)$
  - $w_i$ wage for labor of country $i$, $i = 1, 2, \ldots, M$.
  - $p_j$ price of commodity $j$, $j = 1, 2, \ldots, N$.

- Admissible value $\mathbf{v} = (\mathbf{w}, \mathbf{p}) > 0$:
  No $(i,j) w_i a_{ij} < p_j$ (No production with extraordinary profits)

- Competitive pattern $\mathbf{t} = \{(i, j) \mid w_i a_{ij} = p_j\}$
Main theorem

- At each facet of PP set there exists an admissible international value \( \mathbf{v} = (\mathbf{w}, \mathbf{p}) \) with \( \mathbf{p} \) that is perpendicular to the facet and satisfies equality:

\[
\langle \mathbf{w}, \mathbf{q} \rangle = \langle \mathbf{p}, \mathbf{y} \rangle
\]

where \( \mathbf{y} \) is a point in the facet.

- Competitive pattern of a facet is spanning. The converse is true.
A Minimal Model of the Ricardian Trade Theory

(2 country 3 product case)

v2 Domain 2

\{11, 13, 22, 23\} (A13 B23)

Domain 3 v3

\{11, 12, 13, 22\} (A123 B2)

Domain1 v1

\{11, 21, 22, 23\} (A1 B123)
Wage simplex

If we shift to wage simplex:

Segment that connects two points $w_1$ and $w_3$:

Originally enigmatic!
A spanning core in a wage simplex: (3,3) Ricardian economy case
Wage simplex

Each domain has a different competitive type.

This is in reality a variant of tropical hyperplane arrangement.

Country: (1, 2, 3)
Commodity: (a, c, b)
An arrangement in $\mathbb{TP}^2$

Figure 1 (Ardila & Develin 2004, p.3)

(3,3,3)  (23,13,3)  2  (2,123,3)

(3,1,3)  (123,1,3)  (2,2,3)  (2,2,123)

(1,1,3)  (12,1,13)  (2,12,13)  (2,2,123)

(1,1,1)  (2,1,1)  (2,2,1)  (2,2,2)
Why subtropical algebra?

- **Subtropical semiring**
  - \( a \oplus b = \min\{a, b\} \)  \( a \odot b = a \cdot b \)

- **Ricardian trade theory**
  - Minimum, multiplication (value relations)
  - Minkowski sum (quantity relations)
  - A natural object for (sub)tropical analysis
  - A concrete object for duality

- **Matrix operation**
  - \( w \boxtimes A = \min_i w_i a_{ij} \) is comparable with \( p_j \)
  - \( v \) is admissible \( \iff w \boxtimes A = p \)
Some new ideas (in economics)

• **What happens in the interior of PPS?**
  - Economically, this is to investigate unemployment.
  - This requires study admissible value independent of production point.
  - **Normal value** (main theorem, spanning type)

• **Tropical oriented matroid:**
  - a set of fine types (⇔competitive types)
Necessary labor set

- A and d are given;
- \( L = \{ q_i = (\sum_j s_{ij} a_{ij})_i, \sum_i s_{ij} = d_j \} \)
- An admissible value gives upper facet.
- An anti-admissible value gives lower facet. \( w_{ij} a_{ij} \leq p_j \ \forall \tau = (i,j) \).
- Other values: mixed value
  - \( \exists l,j: wij a_{ij} < p_j \) and \( \exists h,k: w_{hk} a_{hk} > p_k \)
\begin{tabular}{|c|c|c|c|}
\hline
q. of labor & good 1 & good 2 & good 3 \\
\hline
Country A & uA & 1 & 2 & 1 \\
County B & uB & 3 & 1 & 1 \\
Total production & & 1 & 2 & 3 \\
\hline
\end{tabular}
Spanning type determines value.

- $A = (a_{ij})$ is given.
- $v = (w, p) \Rightarrow T = \{\tau=(i,j) | w_i a_{ij} = p_j\}$
- $T$: $(M,N)$ bipartite graph $T \in K_{M,N}$
- $T$: spanning tree
  - connected (tree: one connected component)
  - spanning (edges cover all countries and goods)
  - no cycle (no cyclic chain of edges)
  - In (2,3) trade economy, there are 12 different spanning trees. See the next sheet.
Properties of spanning trees and value determination

- $(M,N)$ spanning tree has $M+N-1$ edges.
- Contains leaves (vertex with degree 1)
- Start by any value from a vertex of a leaf $w_i$ if country vertex $i$ and $p_j$ if product vertex $j$.
- Continue fixing the value of a new vertex by eq. $w_i a_{ij} = p_j$ when $(i,j) \in T$.
- All vertices are covered (spanning) and no contradiction (no cycle)
Matrix $A$ in a general position

- Pallaschke and Rosenmüller (2004), $\mathcal{E} = \{A, q\}$ as a cephoid.
  - Cephoid is a PP set for a Ricardian trade economy.
  - Definition 1.5 ("nondegenerate" or "in general" position) is rather complicated.

- A new definition:
  - $A$ is in a general position $\iff$ $T = \{(i,j) \mid w_i a_{ij} = p_j\}$ is acyclic $\forall w, p$.
  - We may restrict the range of definition to normal values.
Bipartite graph corresponding to directed 2,3 Ricardian trade economy $K_{2,3}$

An example of closed cycle: A2B3A
An acyclicity theorem

Theorem:

If $T_1$ and $T_2$ are two different competitive types of a matrix $A$ in general position, then directed bipartite graph $T_1 \cup T_2$ has no directed cycle.

Proof: Let $v_1$ and $v_2$ be values determined respectively by $T_1$ and $T_2$. If cycle exists, $v_1 = v_2$ or matrix $A$ is not in general position. QED.
Types that can be consistent
Problems:

● Number of spanning trees for bipartite graph \( M^{N-1} \cdot N^{M-1} \) (Scoin’s formula).

● Set of normal types
  - (A1 B123), (A13 B23), (A123 B2)
  - Number of consistent types: equals to the number of multi sets \( H^M_{N-1} = (M+N-2)!/(M-1)! \cdot (N-1)! \)

● Can we characterize the set of normal types that may corresponds to a matrix?

● How many spanning types in a given class?
Really challenging problems:

- Can we extend the theory to RS trade economy? (RS is much more important than R)

Value relation:
\[
\min \{ w_{i0} + a_{i1} p_1 + \cdots + a_{iN} p_N \} = p_N
\]

Tropical parallelism:
\[
\oplus \left\{ w_{i0} \odot p_1^{a_{i1}} \odot \cdots \odot p_N^{a_{iN}} \right\} = p_N
\]

Here
\[ a_{ij} \] can be assumed integral, but very large.
References:

- Shiozawa (2012) Subtropical convex geometry as the Ricardian theory of international trade, in RG.
- Shiozawa (2016) New Interpretation of Ricardo’s four Magic Numbers and the New theory of International Values, in RG.
Thank you.

- Questions and comments welcome.
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