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Non-Simultaneous Mark-up Pricing Processes

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The theory of prices has been investigated from two mutually contradictory view-points. The first theory, once dominant in neo-classical economics and in this sense more orthodox than the other, emphasizes the influence of the demand-supply gap over price fluctuations. The second theory stresses the production cost as the commanding factor of price determination. Probably shared by the classical economists such as Ricardo and Marx, this latter theory has been resurrected due to the apparent influence of the structural change in capitalism from a more anarchic stage to a more monopolistic one. The present paper stands upon the second view-point and tries to see how the prices behave when each agent, industrialist or worker, realizes his desired price or wage at one time or another.

Studies based on the full cost principle, which is the "modern" name given to the production cost analysis, nowadays are very numerous (and the author refrains from citing them); all such studies have a tacit understanding that prices are ajusted all at once for each period. We may call this price adjustment the *simultaneous* pricing assumption. Imposed rather by technical reasons in mathematics than by actual observation, the assumption is evidently a first but unskilled approximation of reality. There may be concerted price raising involving several industries, but this is rarely the case. Usually new prices are announced consecutively one after another. The aim of this paper is to fill in the gap between the existing theory and the simple evidence by introducing the *non-simultaneous* mark-up pricing processes.

1. Definition of the processes

The economy considered is of the Leontief type. There are n-1 commodities, each of which is produced by a different industry. We suppose there is no product differentiation and assume that firms in the same industry behave similarly in price adjustment. The reader may imagine for each industry a price-leader or a 100 per cent monopolist. The production takes place in one period, input composed of labour and commodities being made at the beginning and output available at the end of each period. The combination of input and output in each industry is unique up to scalar multiplication that means that the technique is ruled by constant returns to scale. Then the technology can be summarized by an $(n-1) \times n$ coefficient matrix $C = [\, c_i^j \,] \geqq 0$ $(i=1, \cdots, n;\ j=2, \cdots, n)$, where c_i^j is the amount of the i-th commodity required for one unit of output of the j-th commodity, labour force being counted as the first commodity preceding any other commodity-goods. We assume labour is directly or indirectly necessary for production of any commodity except for the labour force itself, for which we assume the worker has a standard minimum expressed by a basket c^1 a non-generated non-negative line vector of dimension n. Adding c^1 as the first line to the matrix C, we obtain a non-negative square matrix which we express by the same symbol C. There is no confusion, for in the sequel the symbol C uniquely stands for the square matrix. In this way, the j-th line of the matrix C is written c^j irrespectively for any j, either $j = 1$ or $j \geqq 2$.

Let p be a price vector expressed by a suitable numeraire, say a paper money or, in other words, pure exchange means which appear in no industry either as output or input. It is a positive column vector (p^j) of dimension n where p^1 is the monetary wage rate and p^j $(j \geqq 2)$ is the unit price of the j-th commodity. If we write $\langle x , y \rangle$ the scalar product of a line vector x and a column vector y of the same dimension in order to emphasize that the product is a scalar, the worker's demand for the wage rate, given the actual price vector p, is $\langle c^1 , p \rangle$. The real cost for the j-th industrialist is $\langle c^j , p \rangle$ per unit of commodity. But this is not a satisfactory price for him because he expects to gain a *normal* profit from his activity. Let r_j be the mark-up rate of the j-th industry, and then the satisfactory price for his product is $(1 + r_j)\langle c^j , p \rangle$. We don't ask here what determines r_j but suppose they are given once and for all. Let R be a diagonal matrix of dimension n whose j-th diagonal element is $1 + r_j$ for $j \geqq 2$ and 1 for $j = 1$. We are free to take R whose first diagonal element d is different from 1. But it is a mere change of interpretation, for we can take $d \cdot c^1$ from the very first as the standard minimum for the workers. Finally, let $A = RC$. The matrix A expresses the price aspiration of all agents. If A is known and for any price vector p given, one can calculate each agent's demand for price adjustment. In fact, the j-th agent asks to realize $\langle a^j , p \rangle$ for his commodity sale price, where a^j is the j-th line of the matrix A.

Price adjustment takes place ramdomly in time. Yet if we consider that our economic time proceeds only by one unit every time an adjustment takes place, there is no loss of generality in supposing that adjustment occurs only at a time indexed by a natural number t. Nothing necessitates that this unit be equal to the production period. Furthermore, measured by physical time, the unit may well vary from time to time.

Suppose at time t a group of agents succeed in imposing their demanded prices whereas the rest of the agents maintain the same prices. Suppose the price vector is changed over from p(t) to p(t+1) by this adjustment. One can easily calculate p(t+1) from p(t) and A, if one knows the group of successful agents. Thus a non-simultaneous price adjustment *process* is assigned to a sequence of subsets of the set of all agents $\phi(t)$. Let $N = \{1, 2, \cdots, n\}$ and 2^N be the set of all subsets of N. The set of all natural numbers including 0 is written $\mathbb{N}$. Formally, ϕ is a function from $\mathbb{N}$ to 2^N. Simultaneous process is a special case of non-simultaneous processes where $\phi(t) = N$ for all t.

Two kinds of price adjustment can be discerned in the modes of price fluctuations. In the first mode, the prices are downwardly flexible. If the normally desirable price in a commodity market is lower than the current price, the industrialist takes the initiative in an attempt to develop his business, or accepts being forced by their clients to fix the price down to the normal price. In the second mode, which is more acceptable in the monopolist market, the price will be adjusted when the normal price is higher than the current price but will be kept invariant in the other case. This downward inflexibility was often pointed out as one of the main cause of our inflationist economy, although the expression is not very exact, as we shall see later. In this paper, only *inflexible* (needless to say downwardly inflexible and upwardly flexible) adjustments are examined and flexible cases are left for brief remarks which may help to show the differences between the two modes of adjustment.

Let an initial price vector $p(0)$, and aspiration matrix A and an adjustment process ϕ be given. The adjusted price vector at time $t + 1$ is given inductively by the formula $p^j(t + 1) = \max\{p^j(t), \langle a^j, p(t)\rangle\}$ for all j included in $\phi(t)$ and $p^j(t + 1) = p^j(t)$ for the rest of indices. In order to study flexible adjustment, it suffices to replace the formula for the indices in $\phi(t)$ by a simpler formula $p^j(t + 1) = \langle a^j, p(t)\rangle$. In the inflexible adjustment, the fact that an industry belongs to the adjustment group $\phi(t)$ does not make any difference when in the market the normally desired price is lower than the current price. Omit such indices from the set $\phi(t)$ and name the new set $\phi^*(t)$. Inflexible adjustments $\phi(t)$ and $\phi^*(t)$ give the same results $p(t + 1)$. In the naive expression, it is natural to say that price adjustments take place only in commodity markets belonging to $\phi^*(t)$. But this naive setting complicates the study beyond necessity, for $\phi^*(t)$ depends on the current price vector $p(t)$. So we continue to appoint $\phi(t)$ independently of the prices as a possible commodity group of price adjustment and consider that the price *is* in fact adjusted for the commodity which belongs to $\phi(t)$, including cases of non-effective adjustment.

The convention made above is partly justified by the fact that $\phi(t)$ is chosen for various reasons such as the demand-supply gap, market structure, bilateral negotiation power etc., which do not necessarily reflect in a direct form the price position of the commodity concerned. If the market is tight for a commodity j, the industrialist has a good reason to readjust his product price on this occasion, say a time t. So one can include j in $\phi(t)$. But if $p^j(t)$ is higher than his calculation of the normal price, the adjustment remains latent unless he makes up his mind to review the normal rate of profit.

We shall ask what will eventually happen to the prices if an infinite number of adjustments take place according to a predesigned adjustment process. Two cases can be distinguished: the consistent case and the inconsistent case. In the first *consistent* case, there exists a positive price vector p such that no agent feels it necessary to revise his price i.e. $Ap = p$. Otherwise, the aspiration is *inconsistent*. Theoretically, there are two possibilities according to whether the maximal eigenvalue of the matrix A is greater or smaller than 1. The latter possibility can be neglected as a case of no importance, for in such a case after a lapse of finite time the prices can only go down if the adjustment is flexible, and cease to move if the adjustment is inflexible. Anyway, if one wants to investigate this case, one can do similar observations as in section 3. The consistent case will be studied in section 2. The main result of that section is the fact that prices converge to an equilibrium if in the process adjustment takes place an infinite number of times for any commodity. Section 3 is devoted to the study of the inconsistent case, and illustrates in part a mechanism of inflation.

2. The consistent case

By definition, there exists a positive price vector p satisfying the equation $Ap = p$ when the demands of all agents are consistent. Let P be the diagonal matrix formed by the elements of the vector p or, more precisely, the j-th diagonal element of P is the j-th coordinate of the vector p. As p is positive, the matrix P is invertible. The inverse of P is also diagonal and is composed of elements of the form $1/p^j$. By the transformation $P^{-1}AP$, the matrix becomes a stochastic one as it is easy to see that $P^{-1}AP\,\mathbb{1} = \mathbb{1}$ where $\mathbb{1}$ is a vector whose entries are all 1. From now on in this section, we assume that the transformation has already been performed and write the transformed matrix by the same symbol A. Likewise, price vectors incur the same coordinate transformation and the vector $\mathbb{1}$ and its multiples are now equilibrium price vectors.

It would be opportune here to remark that any non-negative vector p of the form $Ap = p$ is unique up to scalar multiplication, and thus is a multiple of the vector $\mathbb{1}$. In fact, when restricted to basic commodities (in the sense of Sraffa), p is proportional to $\mathbb{1}$ from the uniqueness of a non-negative eigen-vector for an irreducible non-negative matrix. As for non-basic commodities, as we have assued that labour is directly or indirectly necessary for the production of any commodity (possibly with an exception of the labour forces), the prices are uniquely determined by the prices of basic commodities.

Take a process ϕ and assume that adjustments take place an infinite number of times for any commodity. The assumption is equivalent to saying that for any index j and for any number T there exists among natural numbers not smaller than T a time t which satisfies the condition $j \in \phi(t)$. Suppose we are given a sequence of indices $j_1, j_2, \cdots, j_s$ and a number T. Starting from t_1 which satisfies $t_1 \geqq T$ and $j_1 \in \phi(t_1)$, one can continue to choose t_i in such a way that $t_i > t_{i-1}$ and $j_i \in \phi(t_i)$ until one arrives at t_s. Thus, for any sequence of indices, one can choose a strictly increasing sequence of times $t_1, t_2, \cdots, t_s$ in any half-line $t \geqq T$ in such a way that each set of price adjustments $\phi(t_i)$ includes the given index j_i in the same position in the index sequence as t_i in the time sequence.

Let an initial price vector $p(0) = p$ be given. A process ϕ together with A gives rise to a well-defined sequence of price vectors p(t). If A is consistent and adjustment is inflexible, p(t) is convergent for any process ϕ. If further ϕ satisfies the condition mentioned in the preceding paragraph, then p(t) converges to a vector of the form $\alpha \cdot \mathbb{1}$ when the vectors are restricted to basic commodities. The first assertion is easy to see. It is well known that, in the case of vectors as in the case of numbers, any non-decreasing sequence is convergent if it is bounded from above. As the adjustment is downwardly inflexible, the sequence $p(t)$ is non-decreasing. The fact that $p(t)$ is bounded from above is a result of the following. A is consistent and we have assumed that A has already been transformed by the matrix P. Therefore, it satisfies the formula $\sum_{i=1}^{n} a_i^j = 1$ for all j. Let $p_*^j(t) = \sum_{i=1}^{n} a_i^j \cdot p^i(t)$. It is a barycentric mean of the numbers $p^i(t)$ and therefore it is not greater than the maximum of $p^i(t)$. As $p^j(t + 1)$ is equal either to $p^j(t)$ or to $p_*^j(t)$, we have for all j, $p^j(t + 1) \leqq \max_{i=1, \ldots, n} p^i(t)$. This is true for any t. By induction we have for any t and j, $p^j(t) \leqq M$ where M is the maximum of the initial values p^i. Thus, the first assertion is verified. As a corollary, it follows that if $p^j(T)$ is maximal among the values $p^i(T)$, then $p^j(t)$ remains constant and maximal for all $t \geqq T$.

The second assertion is more difficult and touches on the detail of non-negative irreducible matrices. To recall the definition of a basic commodity, it is one which enters directly or indirectly into the production of all commodities. As we have supposed that the labour is necessary directly or indirectly to the production of any commodity, and that a^1 is semi-positive, there exists at least one basic commodity other than the labour force. Indeed, any commodity which has a positive co-ordinate in the vector a^1 is basic, for it enters into the labour force and then into the production of all commodities. The collection of all basic commodities gives rise to a sub-economy in the sense that they form a self-sufficient production system. The corresponding production coefficient matrix is a principal sub-matrix of that of the whole economy.

It is irreducible because no sub-division of commodities into two disjoint groups makes one group independent of the other. The prices of non-basic commodities do not affect those of basic commodities, for the former commodities do not enter into the production of the latter ones. Thus, if we restrict ourselves to the basic commodities, we can suppose that A itself is irreducible. Let $D(t) = \max_i p^i(t) - \min_i p^i(t)$. $D(t)$ is non-increasing for the $\max_i p^i(t)$ remains invariant whereas $\min_i p^i(t)$ has an opportunity to become greater as time passes. In fact, it will be proved that $D(t)$ converges to 0 when t tends to infinity.

Take any two indices i and j. If A is irreducible, there is a chain of indices $j = i(0), i(1), \cdots, i(h) = i$ such that $a^{i(s)}_{i(s-1)} > 0$ for all $s = 1, \cdots, h$. If further ϕ contains an infinite number of times any of all indices, one can find for any T an increasing sequence of time $t_0, t_1, \cdots, t_h$ $(T \leqq t_0 < t_1 < \cdots < t_h)$ such that $\phi(t_s)$ contains $i(s)$ for all s. Let j be an index such that $p^j(T)$ is the maximum of $p^i(T)$ when i varies for all indices. For any index i, let us take a chain $j = i(0), \cdots, i(h) = i$ as above. Then, inductively, we can prove that

$$p^j(t_s + 1) - p^i(t_s + 1) \leqq (1 - a^i_{i(s-1)} \cdot \cdots \cdot a^{i(1)}_j) \cdot D(T).$$

Indeed, if the estimate is true for $s - 1$, then

$$p^j(t_s + 1) - p^i(t_s + 1)$$

$$\leqq \sum_{k=1}^n a^{i(s)}_k \, p^j(t_s) - \sum_{k=1}^n a^{i(s)}_k \, p^k(t_s)$$

$$\leqq a_1^{i(s)} \{p^j(t_s) - p^1(t_s)\} + \cdots + a_{i(s-1)}^{i(s)} \cdot \{p^j(t_s) - p^{i(s-1)}(t_s)\} +$$

$$+ \cdots + a_n^{i(s)} \cdot \{p^j(t_s) - p^n(t_s)\}$$

$$\leqq a_1^{i(s)} \cdot D(T) + \cdots + a_{i(s-1)}^{i(s)} \cdot \{1 - a_{i(s-2)}^{i(s-1)} \cdot \ \cdots \ \cdot a_{i(0)}^{i(1)}\} D(T) + \cdots + a_n^{i(s)} \cdot D(T)$$

$$= \{1 - a_{i(s-1)}^{i(s)} \cdot a_{i(s-2)}^{i(s-1)} \cdot \ \cdots \ \cdot a_{i(0)}^{i(1)}\} \cdot D(T).$$

Here we have used the facts that $p^j(t) - p^k(t) \leqq D(T)$ for all $t \geqq T$ and $p^k(t)$ is non-decreasing. The result means that for any sufficiently large t we have $D(t) \leqq (1 - \varepsilon) D(T)$ where ε is the minimum of the positive chain products from j to i when i varies for all indices. Starting from $T_0 = 0$, choose T_s inductively such that $D(T_N) \leqq (1-\varepsilon) D(T_{s-1})$. From this construction, it is easy to see that $D(T_N) \leqq (1-\varepsilon)^N D(0)$. As one can take an N as large as possible, $D(T_N)$ and *a fortiori* $D(t)$ converge to 0 when t tends to infinity.

We have thus proved the convergence of prices in the range of basic commodities provided that the process satisfies a certain repetitive condition for all indices. It would be interesting to see what would happen for the prices of non-basic commodities. For this purpose, observe that the commodities can be grouped into classes by their relation E defined as follows. Any commodity is in relation E with itself. Let i and u be two different commodities. If i enters into the production of j and inversely j enters into that of i, commodities i and j are defined to be in relation E. All of the basic commodities form one class. If a commodity of a class H enters into the production of a commodity of a class G different from H, then no commodity of G enters into the production of any commodity in H. In this case, we say that the class H precedes the class G, or G is dependent on H. Between two different classes, there are only two possibilities. They have no relation of dependence at all, or one of the two is dependent on the other. The class of basic commodities precedes any other classes. Now, we can state how the prices converge when the process ϕ contains any index an infinite number of times. Let H be an equivalence class. Suppose that for all classes G which precede H the limit vectors $u^G = \lim_{t\to\infty} p^G(t)$ are defined. Here, $p^G(t)$ is the restriction on G of the vector p(t). Then there exists a unique vector u_*^H which satisfies the equation $\sum_{G<H} A_G^H \cdot u^G + A_H^H \cdot u_*^H = u_*^H$. Symbols like A_G^H stand for the submatrices whose indices are restricted to the indicated subsets of N. $G < H$ means that G precedes H. If $p^H(0) \leqq u_*^H$, then the limit vector $u^H = \lim_{t\to\infty} p^H(t)$ is equal to u_*^H. If there exists an index $j \in H$ such that $p^j(0) > u_*^j$ then $u^H \geqq u_*^H$. In particular, if $p^j(0) \leqq \alpha$ for all j where α is the maximum of $p^i(0)$ when i varies for all basic commodities, then the limit vector is equal to the equilibrium vector $\alpha \cdot \mathbb{1}$ for all commodities. This demonstration is only a variant of the proof given in the case of basic commodities.

Let us close this section by a brief comment on the case of flexible price adjustments. Although prices remain in a bounded closed interval of positive numbers, convergence is not always assured even in the range of basic commodities. If a non-negative irreduciable martix

A has a period greater than 1, it is well known that simultaneous price adjustment gives rise to a periodic price movement for a suitably chosen initial price vector. Here is an example in which the adjustment process is not simultaneous. Let the matrix A and the initial price vector p be given by the following:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \qquad p = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

Suppose that the adjustment set $\phi(t)$ contains only one index for any t and let it be given by a periodic sequence 3 1 2 3 1 2 $\cdots$. The period is three and ϕ contains any index an infinite number of times. The sequence of price vectors $p(t)$ is also periodic from $p(1)$ on, as follows:

$$\begin{bmatrix} 1 & 1 & 2 & 2 & 2 & 1 & 1 & 1 & \cdots \\ 2 & 2 & 2 & 1 & 1 & 1 & 2 & 2 & \cdots \\ 3 & 1 & 1 & 1 & 2 & 2 & 2 & 1 & \cdots \end{bmatrix}.$$

Each column represents a price vector $p(t)$. The second and eighth columns are identical, and we have obtained the periodicity with the period in this case six.

2. The inconsistent case

Now we shall study the case when the maximal eigenvalue of A is greater then 1. An economically interesting case is one in which there exists a positive vector u such that $Au = \lambda \cdot u$ for a suitable λ. λ is inevitably the largest of all positive eigenvalues and thus greater than 1. It is called the *inconsistency level*. In particular, if A_b is the principal submatrix of A corresponding to the class of basic commodities, λ is the Frobenius eigenvalue of A_b. In the following, we always assume the existence of such a vector u.

It is convenient to take a standard coordinate base such that, expressed by this base, the aspiration matrix satisfies the equation $A \cdot \mathbb{1} = \lambda \cdot \mathbb{1}$. As was done in section 1, this is equivalent to making a transformation $U^{-1} AU$ where U is the diagonal matrix composed of coordinates of the vector u. If we rewrite the equation above for each element, we obtain $\sum_{j=1}^{n} a_j^k = \lambda$ for all k. If $p(0) = \mathbb{1}$ and $\phi(0) \ni j$, then $p^j(1) \geqq \lambda$. Similarly, if $p(0) = \mathbb{1}$ and $\bigcup_{k=0}^{s} \phi(k) = N$, then $p(s + 1) \geqq \lambda \cdot \mathbb{1}$. This is true for any kind of process whether it is flexible or not. For any given ϕ and for any t, order is preserved in the mapping from $p(0)$ to $p(t)$ in the sense that if $p(0) \geqq p'(0)$ then $p(t) \geqq p'(t)$ where $p(t)$ and $p'(t)$ correspond to the adjusted vector of $p(0)$ and $p'(0)$ respectively. So, if $p(0) \geqq \mathbb{1}$ then $p(s + 1) \geqq \lambda \cdot \mathbb{1}$ if the sets $\phi(k)$ from $k = 0$ to s contains all indices at least one time.

Any interval $[s , t)$ with s and t integers is called elementary, when $\bigcup_{k=s}^{t-1} (k) = N$ but not so for any smaller interval contained in $[s , t)$. If the process is simultaneous, any interval of length 1 is elementary. If ϕ contains any index an infinite number of times, then for any natural number s, there is a unique t for which [s , t) becomes elementary. The maximal number of disjoined elementary intervals included in [0 , T) is said to be the pitch of the process at time T and will be written $\pi(T)$. From the observations in the preceding paragraph, it is easy to see that $p(T) \geqq \beta \cdot \lambda^{\pi(T)}$, where β is the minimum of $p^i(0)$. Thus the prices will be inflated at least by the rate $\lambda-1$ each time an elementary interval passes, or in other words, each time the pitch of the process proceeds by 1. The same treatment is possible when we restrict ourselves to the basic commodities. In this case, the pitch proceeds in general faster than the pitch for all commodities. One may thus obtain a better estimation for the prices of basic commodities. Furthermore, if there is a group of basic commodities G such that the principal submatrix of A over G has a maximal eigenvalue greater than 1, then inflation occurs inside of this group even if the prices of other commodities remain stable.

To estimate p(t) from above is more complicated than to estimate it from below. Let, for example, $a_i^i > \varepsilon > 0$ for all i. If the process ϕ repeats the same adjustment two times, that is, if $\phi(2t) = \phi(2t + 1)$ for all t, then one obtains an estimate $p(t) \geqq \beta \cdot [\lambda + \varepsilon(\lambda - 1)]^{\pi(t)} \cdot \mathbb{1}$. Thus it is impossible to estimate p(t) from above by the maximal eigenvalue raised to the power $\pi(t)$. In the following is an upper estimation although it may not be the best possible.

For simplicity, let us consider the case of irreducible A. Assume there is no proper subset of indices G such that the pricipal submatrix restricted to G has a maximal eigenvalue not smaller than 1. Let $A(k , \rho)$ be the matrix formed from A by replacing the k-th line by $\rho^{-1} a^k$. There is a unique $\rho = \rho_k$ such that $A(k , \rho)$ is a consistent matrix. From the strict monotonicity of Frobenius eivengalues among irreducible matrices, the uniqueness is trivial. Further, one has $\rho > \lambda$. The existence of ρ is assured by the fact that the matrix A, when their k-th line and column are omitted, has a maximal eivenvalue smaller than 1. Now, if ρ is uniquely determined, define u_k to be a positive vector which satisfies $A(k , \rho)\, u_k = u_k$. The vector u_k is unique up to the multiplication by scalars. Then we define a number g called the gauge by the following formula:

$$g = \min_{\alpha=(\alpha_1, \ldots, \alpha_n)>0} \max_{i,j,k} (\alpha_k u_k^i / \alpha_j u_j^i)$$

This number is independent of the choice of u_k's, so it has a geometric meaning inherent in the system of n positive directions. One can suppose that u_k's themselves give the minimum g. Then, $u_k \leqq g \cdot u_j$ for any couple (k , j). Let k be the last index which appears in the first elementary interval $[0 , t)$ and suppose $p(0) \leqq \alpha \cdot u_k$ for a certain α. Then, $p(s) \leqq \alpha\, u_k$ if $s = t - 1$. Indeed, except for $s = t - 1$, $\phi(s)$ does not contain k. But as $A(k , \rho)$ is identical to A except for the k-th line, the adjustment $\phi(s)$ by A has the same effect as the adjustment by the matrix $A(k , \rho)$. Noting that $A(k , \rho)$ is consistent, the estimate follows for $s = t - 1$. The adjusted price of commodity k at time t can be estimated from above by $\alpha \cdot \rho_k \cdot u_k^k$. It follows then the estimate $p(t) \leqq \alpha \cdot \rho_k \cdot u_k^k \leqq \alpha(\rho \cdot g) \cdot u_j$ for any index j where ρ is defined to be the maximum of ρ_k. One may thus proceed to the next elementary interval, if one replaces α by $\alpha(\rho \cdot g)$. In general, one obtains the result as follows: $p(t) \leqq \beta(\rho \cdot g)^{\pi(t)} \cdot \mathbb{1}$. Here β is chosen so as to satisfy $p(0) \leqq \beta \cdot u_j$ for all j. In particular, $p(t)$ remains bounded if $\phi(t)$ fails to contain an index an infinite number of times. Of course, in this case, it is assumed that A satisfies the condition mentioned at the beginning of this paragraph.

Let us ask another quention. Is there any relative stability of price vectors such that $p(t)$ asymptotically converges to a positive half-line which starts from the origin? The answer is negative for general ϕ, as we shall see in the following. Let us suppose the contrary. The positive half-line must remain unchanged by any transformation of price adjustment. In particular, it is not changed by the transformation A. Then, a vector u of the half line satisfies the equation $Au = \lambda \cdot u$. But if we apply a price adjustment only for an index j, then the adjusted vector cannot be proprotional to u as u^j changes but as others remain unchanged and this is contradictory. We know that the simultaneous price adjustment for a primitive matrix is relatively stable. Furthermore, we can estimate its growth order accurately. We see now that it was only a happy case of relative stability and in general there is not such regular price movement. Let us content ourselves here with the trivial assertion that the price vector $p(t)$ remains strictly positive for any process and for all t.

Up to now, we have not specified if the adjustment concerned was flixible or inflexible. In the inconsistent case, the distinction is indeed only nominal, for even in the flexible case the prices only have a chance to be adjusted upward after a finite time of adjustment. The difference in the two modes of adjustment arises when the initial price vector is given arbitrarily (as a mathematical possibility) and there may be some commodities for which desired prices are lower than current prices. But even in this case, after the first elementary interval of times has passed, there is no commodity whose newly marked up price is lower than the current price. Inflation occurs whether the adjustment is flexible or inflexible. It occurs when and only when the aspirations of all agents are inconsistent. Although in a real world of economy, the main factor of inconsistency can be focused on the contradictory relationship between wage workers and capitalists, it can also be noted that inflation occurs theoretically if some group of industrialists make up an inconsistent system of mark-up rates among themselves. This is the case where there is no ρ_1 such that $A(1, \rho_1)$ for basic commodities has the Frobenius eigenvalue equal to 1. In section 1, I remarked that it is not accurate to say that downward inflexibility is a cause of inflation. In the same sense one may remark that there is no "cost-push-inflation" but there is an inconsistency of demand levels for the commodity prices.

Conclusion.

In section 1, I explained how the non-simultaneous price adjustment processes are defined. The main results of the following two sections can be summarized in the form of two theorems:

Theorem 1. If the demand levels for the commodity prices are consistent, then any downward inflexible price adjustment process converges. Further, if the process contains an infinite number of adjustments of any commodity, then the price vector converges to an equilibrium in the range of basic commodities.

Theorem 2. If the demand levels for the commodity prices are inconsistent, then inflation occurs at least at the rate $\lambda - 1$ when the time is counted by the pitch of the process. It is understood that λ indicates the inconsistency level. As far as the basic commodities are concerned, inflation can be estimated from above by an expotential function of the pitch if there is no inconsistency in the proper subsets of commodities.

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